EXAMPLE 6–12 A steel bar undergoes cyclic loading such that $\sigma_{\max} = 420$ MPa and $\sigma_{\min} = -140$ MPa. For the material, $S_{ut} = 560$ MPa, $S_y = 455$ MPa, a fully corrected endurance limit of $S_e = 280$ MPa, and $f = 0.9$. Estimate the number of cycles to a fatigue failure using:

(*a*) [illegible] Goodman criterion.

(*b*) Gerber criterion.

Solution From the given stresses,

$$\sigma_a = \frac{420-(-140)}{2} = 280 \text{ MPa} \qquad \sigma_m = \frac{420+(-140)}{2} = 140 \text{ MPa}$$

From the material properties, Eqs. (6–14) to (6–16), p. 277, give

$$a = \frac{(fS_{ut})^2}{S_e} = \frac{[0.9(560)]^2}{280} = 907 \text{ MPa}$$

$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{0.9(560)}{280}\right] = -0.0851$$

$$N = \left(\frac{S_f}{a}\right)^{1/b} = \left(\frac{S_f}{907}\right)^{-1/0.0851} \tag{1}$$

where S_f replaced σ_a in Eq. (6–16).

(*a*) The modified Goodman line is given by Eq. (6–45), p. 298, where the endurance limit S_e is used for infinite life. For finite life at $S_f > S_e$, replace S_e with S_f in Eq. (6–45) and rearrange giving

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

$$S_f = \frac{\sigma_a}{1 - \dfrac{\sigma_m}{S_{ut}}} = \frac{280}{1 - \dfrac{140}{560}} = 373 \text{ MPa}$$

Substituting this into Eq. (1) yields

Answer
$$N = \left(\frac{373}{907}\right)^{-1/0.0851} \doteq 3.4(10^4) \text{ cycles}$$

(*b*) For Gerber, similar to part (*a*), from Eq. (6–46),

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

$$S_f = \frac{\sigma_a}{1 - \left(\dfrac{\sigma_m}{S_{ut}}\right)^2} = \frac{280}{1 - \left(\dfrac{140}{280}\right)^2} = 299 \text{ MPa}$$

Again, from Eq. (1),

Answer
$$N = \left(\frac{299}{907}\right)^{-1/0.0851} \doteq 4.6(10^5) \text{ cycles}$$

Comparing the answers, we see a large difference in the results. Again, the modified Goodman criterion is conservative as compared to Gerber for which the moderate difference in S_f is then magnified by a logarithmic S, N relationship.